

# Transverse beam polarization and CP violation in $e^+e^- \rightarrow \gamma Z$ with contact interactions

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## Abstract

We consider the most general gauge-invariant, chirality-conserving contact interactions in the process  $e^+e^- \rightarrow \gamma Z$ , of the type proposed Abraham and Lampe, in order to explore the possibility of CP violation at future linear colliders in the presence of polarized beams. We hereby extend recent work on CP violation due to anomalous triple-gauge boson vertices. We isolate combinations of couplings which are genuinely CP violating, pointing out which of these can only be studied with the use of transverse polarization. We place constraints on these couplings that could arise from suitably defined CP-odd asymmetries, considering realistic polarization (either longitudinal or transverse) of 80% and 60% for the electron and positron beams respectively, and with an integrated luminosity  $\int dt \mathcal{L}$  of  $500 \text{ fb}^{-1}$  at a centre of mass energy of  $\sqrt{s} = 500 \text{ GeV}$ .

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# 1 Introduction

An  $e^+e^-$  linear collider (LC) operating at a centre-of-mass (cm) energy of several hundred GeV is now a distinct possibility. At such a facility, one would like to determine precisely known interactions, and discover or constrain new interactions. Longitudinal polarization of the  $e^+$  and  $e^-$  beams, which is expected to be feasible at such colliders, would be helpful in reducing background as well as enhancing the sensitivity. Spin rotators can be used to convert the longitudinal polarizations of the beams to transverse polarizations. These developments have led to a series of investigations on the use of transverse polarization in achieving these aims, see, e.g. [1].

One sensitive window to the possibility of observing new physics is through the observation of CP violation in processes where it is expected to be either absent or suppressed in the standard model. In the context of CP violation, the role of transverse polarization has been studied in [2, 3, 4, 5, 6], whereas that of longitudinal polarization in [7, 8], and in references quoted therein. The potential of longitudinal polarization to improve the sensitivity of CP-violating observables has been known for a long time. The transverse polarization potential at the LC was recently proposed in the context of  $t\bar{t}$  production [4], where the need for chirality violating interactions for the observation of CP violation through top azimuthal distribution was emphasized. In case of a neutral final state, however, CP violation is possible to observe even with chirality conserving interactions. In  $\gamma Z$  production a CP-violating contribution can arise if anomalous CP-violating  $\gamma\gamma Z$  and  $\gamma ZZ$  couplings are present [9, 10]. The interference of the contributions from these anomalous couplings with the SM contribution give rise to the polar-angle forward-backward asymmetry with unpolarized [9] or longitudinally polarized beams [10], as well as new combinations of polar and azimuthal asymmetries in the presence of transversely polarized beams [5].

However, there may be sources different from anomalous triple-gauge-boson vertices that could also contribute to such asymmetries. A set of model-independent form factors that are gauge invariant and chirality conserving were proposed as such sources in ref.[11] in the context of  $Z \rightarrow b\bar{b}\gamma$  events. It is the purpose of this work to make use of such general form factors for the process  $e^+e^- \rightarrow \gamma Z$  to examine CP-violating asymmetries in the presence of longitudinal or transverse polarization. Our emphasis will however be on transverse polarization, since it provides a handle on a different and larger set of form factors, as will be seen below. We employ these form factors and evaluate their contribution to the differential cross section and pertinent asymmetries to leading order.

In general, these form factors can be functions of both  $s$  and  $t$ ; here we consider the dependence on  $t$  to be absent and treat them as constants at a fixed  $\sqrt{s}$ . The analysis would be considerably more complicated if we put in the dependence of form factors on  $t$  as well.

Closely related sources of CP violation have been constrained experimentally at the LEP collider [12] in the reaction  $Z \rightarrow b\bar{b}g$  (see also [13]) and have been considered elsewhere [14, 15].

In Sec.2 we describe the form factors for the process of interest and compute the differ-

ential cross section due to the SM and the anomalous couplings, the latter to leading order. In Sec.3 we describe the construction of CP-odd asymmetries from which we can extract the anomalous couplings and provide a detailed discussion on their utility, followed by numerical results in Sec.4. We find that the different anomalous couplings can be constrained at a realistic LC with design luminosities of  $500 \text{ fb}^{-1}$  at varying levels, lying between  $10^{-4} - 10^{-2}$ . In Sec.5 we summarize our conclusions.

## 2 The process $e^+e^- \rightarrow \gamma Z$ with anomalous form factors

The process considered is

$$e^-(p_-, s_-) + e^+(p_+, s_+) \rightarrow \gamma(k_1, \alpha) + Z(k_2, \beta). \quad (1)$$

We shall assume that the amplitudes are generated by the standard model as well as a general set of CP-violating interactions of the type proposed by Abraham and Lampe [11]. They are completely determined by vertex factors that we denote by  $\Gamma_{\alpha\beta}^{SM}$  and  $\Gamma_{\alpha\beta}$ . The vertex factor corresponding to SM is given by

$$\Gamma_{\alpha\beta}^{SM} = \frac{e^2}{4 \sin \theta_W \cos \theta_W} \left\{ \gamma_\beta (g_V - g_A \gamma_5) \frac{1}{\not{p}_- - \not{k}_1} \gamma_\alpha + \gamma_\alpha \frac{1}{\not{p}_- - \not{k}_2} \gamma_\beta (g_V - g_A \gamma_5) \right\}. \quad (2)$$

In the above, the vector and axial vector  $Z$  couplings of the electron are

$$g_V = -1 + 4 \sin^2 \theta_W; \quad g_A = -1. \quad (3)$$

The anomalous form factors may be introduced via the following vertex factor:

$$\begin{aligned} \Gamma_{\alpha\beta} = \frac{ie^2}{4 \sin \theta_W \cos \theta_W} \left\{ \frac{1}{m_Z^4} ((v_1 + a_1 \gamma_5) \gamma_\beta (2p_{-\alpha} (p_+ \cdot k_1) - 2p_{+\alpha} (p_- \cdot k_1)) + \right. \\ ((v_2 + a_2 \gamma_5) p_{-\beta} + (v_3 + a_3 \gamma_5) p_{+\beta}) (\gamma_\alpha 2p_- \cdot k_1 - 2p_{-\alpha} \not{k}_1) + \\ ((v_4 + a_4 \gamma_5) p_{-\beta} + (v_5 + a_5 \gamma_5) p_{+\beta}) (\gamma_\alpha 2p_+ \cdot k_1 - 2p_{+\alpha} \not{k}_1) \Big) + \\ \left. \frac{1}{m_Z^2} (v_6 + a_6 \gamma_5) (\gamma_\alpha k_{1\beta} - \not{k}_1 g_{\alpha\beta}) \right\}. \quad (4) \end{aligned}$$

This is the most general form of coupling consistent with Lorentz invariance, gauge invariance and chirality conservation. These couplings include contact interactions, as well as contributions from triple gauge vertices considered in [10, 5]. The latter would be a special case of our general interactions. We note here that not all the form factors are CP violating. The following combinations are CP odd:  $r_2 + r_5, r_3 + r_4, r_6$ ;  $r = v, a$ . The combinations  $r_1, r_2 - r_5, r_3 - r_4$ ;  $r = v, a$ , are even under CP.

When the  $e^-$  and  $e^+$  beams have longitudinal polarizations  $P_L$  and  $\overline{P}_L$ , we obtain the differential cross section for the process (1) to be

$$\frac{d\sigma}{d\Omega_L} = \mathcal{B}_L (1 - P_L \overline{P}_L) \left[ \frac{1}{\sin^2 \theta} \left( 1 + \cos^2 \theta + \frac{4\overline{s}}{(\overline{s} - 1)^2} \right) + C_L \right], \quad (5)$$

where

$$\overline{s} \equiv \frac{s}{m_Z^2}, \quad \mathcal{B}_L = \frac{\alpha^2}{16 \sin^2 \theta_W m_W^2 \overline{s}} \left( 1 - \frac{1}{\overline{s}} \right) (g_V^2 + g_A^2 - 2P g_V g_A), \quad (6)$$

with

$$P = \frac{P_L - \overline{P}_L}{1 - P_L \overline{P}_L}, \quad (7)$$

and

$$C_L = \frac{1}{4(g_V^2 + g_A^2 - 2P g_V g_A)} \left\{ \sum_{i=1}^6 ((g_V - P g_A) \text{Im} v_i + (g_A - P g_V) \text{Im} a_i) X_i \right\}. \quad (8)$$

The differential cross section for transverse polarizations  $P_T$  and  $\overline{P}_T$  of  $e^-$  and  $e^+$  is given by

$$\frac{d\sigma}{d\Omega} = \mathcal{B}_T \left[ \frac{1}{\sin^2 \theta} \left( 1 + \cos^2 \theta + \frac{4\overline{s}}{(\overline{s} - 1)^2} - P_T \overline{P}_T \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2} \sin^2 \theta \cos 2\phi \right) + C_T \right], \quad (9)$$

where  $\overline{s}$  is as before,

$$\mathcal{B}_T = \frac{\alpha^2}{16 \sin^2 \theta_W m_W^2 \overline{s}} \left( 1 - \frac{1}{\overline{s}} \right) (g_V^2 + g_A^2), \quad (10)$$

and

$$C_T = \frac{1}{4(g_V^2 + g_A^2)} \left\{ \sum_{i=1}^6 (g_V \text{Im} v_i + g_A \text{Im} a_i) X_i + P_T \overline{P}_T \sum_{i=1}^6 ((g_V \text{Im} v_i - g_A \text{Im} a_i) \cos 2\phi + (g_A \text{Re} v_i - g_V \text{Re} a_i) \sin 2\phi) Y_i \right\} \quad (11)$$

$X_i, Y_i$  ( $i = 1, \dots, 6$ ) are given in Table 1. In the expressions above  $\theta$  is the angle between photon and the  $e^-$  directions, and  $\phi$  is the azimuthal angle of the photon, with  $e^-$  direction chosen as the  $z$  axis and the direction of its transverse polarization chosen as the  $x$  axis. The  $e^+$  transverse polarization direction is chosen parallel to the  $e^-$  transverse polarization direction.

We have kept only terms of leading order in the anomalous couplings, since they are expected to be small. The above expression may be obtained either by using standard trace

$i$	$X_i$	$Y_i$
1	$-2\bar{s}(\bar{s} + 1)$	0
2	$\bar{s}(\bar{s} - 1)(\cos \theta - 1)$	0
3	0	$\bar{s}(\bar{s} - 1)(\cos \theta - 1)$
4	0	$\bar{s}(\bar{s} - 1)(\cos \theta + 1)$
5	$\bar{s}(\bar{s} - 1)(\cos \theta + 1)$	0
6	$2(\bar{s} - 1) \cos \theta$	$2(\bar{s} - 1) \cos \theta$

Table 1: The contribution of the new couplings to the polarization independent and dependent parts of the cross section

techniques for Dirac spinors with a transverse spin four-vector, or by first calculating helicity amplitudes and then writing transverse polarization states in terms of helicity states. We note that the contribution of the interference between the SM amplitude and the anomalous amplitude vanishes for  $s = m_Z^2$ . The reason for this is that for  $s = m_Z^2$  the photon in the final state is produced with zero energy and momentum, and for the photon four-momentum  $k_1 = 0$ , the anomalous contribution (4) vanishes identically. A noteworthy feature of the result is that with the exception of the case of  $v_6$  and  $a_6$ , the anomalous form factors either contribute to the transverse polarization dependent part, or to the longitudinal polarization dependent and polarization independent parts of the differential cross section, but not both. It is only for the case of  $i = 6$  that the differential cross section receives contribution to both. We note that the results corresponding to the case of the anomalous triple-gauge-boson vertices [5] is reproduced by the choice  $v_i = a_i = 0$  ( $i \neq 6$ ),  $v_6 = (g_V \lambda_1 - \lambda_2)/2$  and  $a_6 = g_A \lambda_1/2$ .

It is also interesting to note that the combination  $r_2 + r_5 + r_3 + r_4$  give the same angular distribution as  $r_6$ , and that the combination  $r_2 - r_5$  gives the same angular distribution as  $r_1$  (with  $r$  standing for  $v$  and  $a$  in both cases). This implies that so far as the angular distribution from the interference terms is concerned, the number of independent form factors is less than what is displayed in eq. (4). In fact, there are only 6 independent quantities that can be determined by the angular distribution, which are the coefficients of the various combinations of trigonometric functions occurring in the angular distribution, of which 3 are CP violating. On the other hand, the number of independent form factors being 12, the number of real parameters it corresponds to is 24. Clearly, not all these can be determined by the angular distribution, but only certain linear combinations. Moreover, so far as the real parts of form factors are concerned, it is only the combinations  $g_A \text{Re} v_i - g_V \text{Re} a_i$  which appear. Thus it is not possible to separately determine the real parts of  $v_i$  and  $a_i$ .

The angular distribution derived above can be used to construct various asymmetries which can isolate CP-conserving as well as CP-violating combinations of form factors. We will however concentrate only on CP-violating form factor in what follows.

### 3 CP-odd asymmetries

We now present a discussion of the possible CP-odd asymmetries in the process.

We first take up the case of transverse polarization. In order to understand the CP properties of various terms in the differential cross section, we note the following relations:

$$\vec{P} \cdot \vec{k}_1 = \frac{\sqrt{s}}{2} |\vec{k}_1| \cos \theta, \quad (12)$$

$$(\vec{P} \times \vec{s}_- \cdot \vec{k}_1)(\vec{s}_+ \cdot \vec{k}_1) + (\vec{P} \times \vec{s}_+ \cdot \vec{k}_1)(\vec{s}_- \cdot \vec{k}_1) = \frac{\sqrt{s}}{2} |\vec{k}_1|^2 \sin^2 \theta \sin 2\phi, \quad (13)$$

$$(\vec{s}_- \cdot \vec{s}_+)(\vec{P} \cdot \vec{P} \vec{k}_1 \cdot \vec{k}_1 - \vec{P} \cdot \vec{k}_1 \vec{P} \cdot \vec{k}_1) - 2(\vec{P} \cdot \vec{P})(\vec{s}_- \cdot \vec{k}_1)(\vec{s}_+ \cdot \vec{k}_1) = -\frac{s}{4} |\vec{k}_1|^2 \sin^2 \theta \cos 2\phi, \quad (14)$$

where  $\vec{P} = \frac{1}{2}(\vec{p}_- - \vec{p}_+)$ , and it is assumed that  $\vec{s}_+ = \vec{s}_-$ . Observing that the vector  $\vec{P}$  is C and P odd, that the photon momentum  $\vec{k}_1$  is C even but P odd, and that the spin vectors  $\vec{s}_\pm$  are P even, and go into each other under C, we can immediately check that only the left-hand side (lhs) of eq. (12) is CP odd, while the lhs of eqs. (13) and (14) are CP even. Of all the above, only the lhs of (13) is odd under naive time reversal T. In the light of the observations above, as well as the general discussion provided in the previous section on the CP properties of (combinations of) the form factors, we note that it is only the coefficients of  $r_2 + r_5$ ,  $r_3 + r_4$ ,  $r_6$ ,  $r = v, a$  that have a pure  $\cos \theta$  dependence. Consequently, the coefficients of the combinations  $r_1$ ,  $r_2 - r_5$ ,  $r_3 - r_4$ ,  $r = v, a$ , have no  $\cos \theta$  dependence. Moreover, invariance under CPT implies that terms with the right-hand side (rhs) of (12) by itself, or multiplying the rhs of (14) would occur with absorptive (imaginary) parts of the form factors, whereas the rhs of (12) multiplied by the rhs of (13) would appear with dispersive (real) parts of the form factors. We will see this explicitly below when we construct asymmetries which isolate the various angular dependences.

For longitudinal polarization, in addition to (12), there is another CP-odd quantity, viz.,

$$\frac{1}{2} (\vec{s}_- + \vec{s}_+) \cdot \vec{k}_1 = |\vec{k}_1| \cos \theta. \quad (15)$$

While this is also proportional to  $\cos \theta$  like (12), it is expected to appear with a factor  $(P_L - \overline{P}_L)$  multiplying it. It is also CPT odd, and would therefore occur with the absorptive parts of form factors.

We now proceed to construct asymmetries of interest and derive the numerical consequences to the anomalous form factors. We begin by noting that we shall assume a cut-off  $\theta_0$  on the polar angle  $\theta$  of the photon in the forward and backward directions. This cut-off is needed to stay away from the beam pipe. It can further be chosen to optimize the sensitivity. The total cross section corresponding to the cut  $\theta_0 < \theta < \pi - \theta_0$  can then be easily obtained by integrating the differential cross section above.

We now define the following CP-odd asymmetries,  $A_1(\theta_0)$ ,  $A_2(\theta_0)$ ,  $A_3(\theta_0)^\dagger$  which combine, in general, a forward-backward asymmetry with an appropriate asymmetry in  $\phi$ , so as to isolate appropriate anomalous couplings:

$$A_1 = \frac{1}{\sigma_0} \sum_{n=0}^3 (-1)^n \left( \int_0^{\cos \theta_0} d \cos \theta - \int_{-\cos \theta_0}^0 d \cos \theta \right) \int_{\pi n/2}^{\pi(n+1)/2} d\phi \frac{d\sigma}{d\Omega}, \quad (16)$$

$$A_2 = \frac{1}{\sigma_0} \sum_{n=0}^3 (-1)^n \left( \int_0^{\cos \theta_0} d \cos \theta - \int_{-\cos \theta_0}^0 d \cos \theta \right) \int_{\pi(2n-1)/4}^{\pi(2n+1)/4} d\phi \frac{d\sigma}{d\Omega}, \quad (17)$$

and

$$A_3(\theta_0) = \frac{1}{\sigma_0} \left( \int_{-\cos \theta_0}^0 d \cos \theta - \int_0^{\cos \theta_0} d \cos \theta \right) \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}, \quad (18)$$

with

$$\sigma_0 \equiv \sigma_0(\theta_0) = \int_{-\cos \theta_0}^{\cos \theta_0} d \cos \theta \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega}. \quad (19)$$

Of the asymmetries above,  $A_1$  and  $A_2$  exist only in the presence of transverse polarization, and are easily evaluated to be

$$A_1(\theta_0) = \mathcal{B}'_T P_T \overline{P}_T [g_A \{ \overline{s}(\text{Rev}_3 + \text{Rev}_4) + 2\text{Rev}_6 \} - g_V \{ \overline{s}(\text{Rea}_3 + \text{Rea}_4) + 2\text{Rea}_6 \}], \quad (20)$$

$$A_2(\theta_0) = \mathcal{B}'_T P_T \overline{P}_T [g_V \{ \overline{s}(\text{Im}v_3 + \text{Im}v_4) + 2\text{Im}v_6 \} - g_A \{ \overline{s}(\text{Im}a_3 + \text{Im}a_4) + 2\text{Im}a_6 \}], \quad (21)$$

In the equations above, we have defined

$$\mathcal{B}'_T = \frac{\mathcal{B}_T(\overline{s} - 1) \cos^2 \theta_0}{(g_V^2 + g_A^2) \sigma_0^T}. \quad (22)$$

with

$$\sigma_0^T = 4\pi \mathcal{B}_T \left[ \left\{ \frac{\overline{s}^2 + 1}{(\overline{s} - 1)^2} \ln \left( \frac{1 + \cos \theta_0}{1 - \cos \theta_0} \right) - \cos \theta_0 \right\} \right]. \quad (23)$$

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<sup>†</sup>Alternatively, with transverse polarization we could use for  $A_3$  the definition of ref.[5] which would receive contributions from both polarization independent and dependent parts of the cross sections. This would then result in  $A_3(\theta_0) = \mathcal{B}'_T \frac{\pi}{2} [g_A \{ \overline{s}(\text{Im}a_2 + \text{Im}a_5) + 2\text{Im}a_6 \} + g_V \{ \overline{s}(\text{Im}v_2 + \text{Im}v_5) + 2\text{Im}v_6 \}] + A_2(\theta_0)$ . With polarization flips, it would then be possible to separate real and imaginary parts of  $r_3 + r_4 + 2r_6/\overline{s}$ , ( $r = v, a$ ), from  $A_1$  and  $A_2$ , and imaginary parts of  $r_2 + r_5 + 2r_6/\overline{s}$ , ( $r = v, a$ ) from this  $A_3$ . With the present definition, however, the role of longitudinal polarization in enhancing the sensitivity of observables is particularly transparent.

The asymmetry  $A_3$  is independent of transverse polarization and is found to be

$$A_3(\theta_0) = \mathcal{B}'_L \frac{\pi}{2} [(g_A - P g_V) \{\bar{s}(\text{Im}a_2 + \text{Im}a_5) + 2\text{Im}a_6\} + (g_V - P g_A) \{\bar{s}(\text{Im}v_2 + \text{Im}v_5) + 2\text{Im}v_6\}], \quad (24)$$

where

$$\mathcal{B}'_L = \frac{\mathcal{B}_L(1 - P_L \bar{P}_L)(\bar{s} - 1) \cos^2 \theta_0}{(g_V^2 + g_A^2 - 2P g_V g_A) \sigma_0^L}. \quad (25)$$

with

$$\sigma_0^L = 4\pi \mathcal{B}_L(1 - P_L \bar{P}_L) \left[ \left\{ \frac{\bar{s}^2 + 1}{(\bar{s} - 1)^2} \ln \left( \frac{1 + \cos \theta_0}{1 - \cos \theta_0} \right) - \cos \theta_0 \right\} \right]. \quad (26)$$

We now make some observations on the above expressions which justify the choice of our asymmetries and highlight the novel features of our work. It can be seen that  $A_1(\theta_0)$  is proportional to combinations of  $\text{Re}v_i$ ,  $\text{Re}a_i$ , and the other two asymmetries depend on combinations of  $\text{Im}v_i$ ,  $\text{Im}a_i$ . Indeed, but for the case  $i = 6$ , one of the latter asymmetries depends on a specific combination of couplings that is complementary to that which shows up in the other. The case of the anomalous triple-gauge-boson vertex is similar to that of the case  $i = 6$  since in this case there are contributions to both the polarization dependent as well as the polarization independent part of the cross section.

## 4 Numerical Results

We have several form factors, and if all of them are present simultaneously, the analysis of numerical results would be complicated. We therefore choose one form factor to be nonzero at a time to discuss numerical results.

We first take up for illustration the case when only  $\text{Re } v_6$  is nonzero, since the results for other CP-violating combinations can be deduced from this case. We choose  $P_T = 0.8$  and  $\bar{P}_T = 0.6$ , and vanishing longitudinal polarization for this case. Fig. 1 shows the asymmetries  $A_i$  as a function of the cut-off when the values of the anomalous couplings  $\text{Re } v_6$  (for the case of  $A_1$ ) and  $\text{Im } v_6$  (for the case of  $A_2$  and  $A_3$ ) alone are set to unity. The asymmetries vanish not only for  $\theta_0 = 0$ , by definition, but also for  $\theta_0 = 90^\circ$ , because they are proportional to  $\cos \theta_0$ . Also, they peak at around  $45^\circ$ .

We have calculated 90% CL limits that can be obtained with a LC with  $\sqrt{s} = 500$  GeV,  $\int L dt = 500 \text{ fb}^{-1}$ ,  $P_T = 0.8$ , and  $\bar{P}_T = 0.6$  making use of the asymmetries  $A_i$  ( $i = 1, 2$ ). For  $A_3$ , we assume unpolarized beams.

The limiting value  $v^{\text{lim}}$  ( *i.e.* the respective real or imaginary part of the coupling) is related to the value  $A$  of the asymmetry for unit value of the coupling constant.

$$v^{\text{lim}} = \frac{1.64}{|A| \sqrt{N_{SM}}}, \quad (27)$$



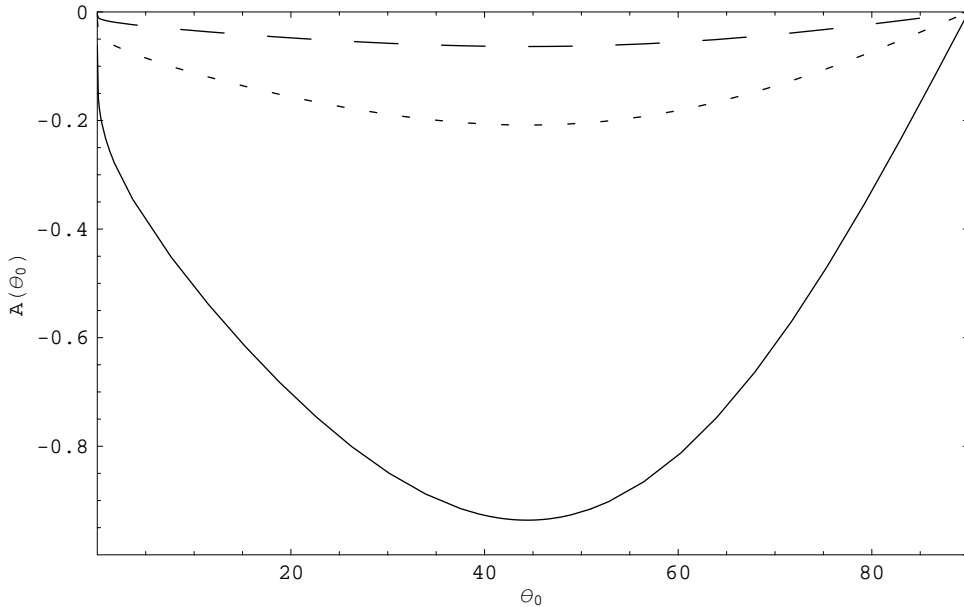


Figure 1: The asymmetries  $A_1(\theta_0)$  (solid line),  $A_2(\theta_0)$  (dashed line) and  $A_3(\theta_0)$  (dotted line), defined in the text, plotted as functions of the cut-off  $\theta_0$  for a value of  $\text{Re } v_6 = \text{Im } v_6 = 1$ .

where  $N_{SM}$  is the number of SM events.

The curves from  $A_1$  corresponding to setting only  $\text{Re } v_6$  nonzero, and from  $A_2$  and  $A_3$  corresponding to keeping only  $\text{Im } v_6$  nonzero are illustrated in Fig. 2. We note that there is a stable plateau for a choice of  $\theta_0$  such that  $10^\circ \lesssim \theta_0 \lesssim 40^\circ$ ; and we choose the optimal value of  $26^\circ$ . The sensitivity corresponding to this for  $\text{Re } v_6$  is  $\sim 3.1 \cdot 10^{-3}$ .

The results for the other couplings may be inferred in a straightforward manner from the explicit example above. For the asymmetry  $A_1$ , if we were to set  $v_3(v_4)$  to unity, with all the other couplings to zero, then the asymmetry would be simply scaled up by a value  $\bar{s}/2$ , which for the case at hand is  $\simeq 14.8$ . The corresponding limiting value would be suppressed by the reciprocal of this factor.

The results for the couplings  $\text{Re } a_i$ ,  $i = 2, 5, 6$ , compared to what we have for the vector couplings would be scaled by a factor  $g_V/g_A \simeq 0.07$  for the asymmetries and by the reciprocal of this factor for the sensitivities.

The results coming out of the asymmetry  $A_2$  are such that the sensitivities of the imaginary parts of  $v$  and  $a$  are interchanged *vis à vis* what we have for the real parts coming out of  $A_1$ .

The final set of results we have is for the form factors that may be analyzed via the asymmetry  $A_3$ , which depends only on longitudinal polarizations. We treat the cases of unpolarized beams and longitudinally polarized beams with  $P_L = 0.8$ , and  $\overline{P}_L = -0.6$  separately. For the unpolarized case, the results here for  $\text{Im } v_6$  correspond to those coming

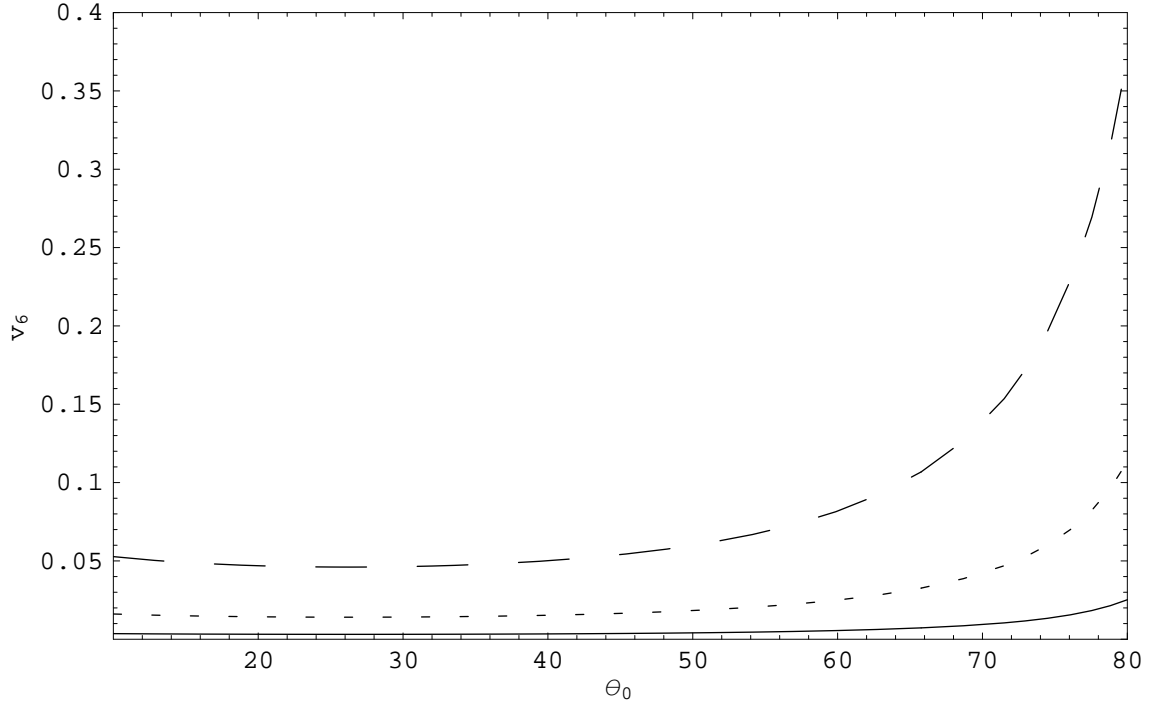


Figure 2: The 90% C.L. limit on  $\text{Re } v_6$  from the asymmetry  $A_1(\theta_0)$  (solid line), and on  $\text{Im } v_6$  from  $A_2(\theta_0)$  (dashed line) and  $A_3(\theta_0)$  (dotted line), plotted as functions of the cut-off  $\theta_0$ .

from  $A_2$ , with the asymmetry scaled up now by a factor corresponding to  $\pi/2$  and a further factor  $(P_T \overline{P}_T)^{-1}$  ( $\simeq 2.1$ ), which yields an overall factor of  $\sim 3.3$ . The corresponding sensitivity is smaller is by the same factor. Indeed, the results we now obtain for  $\text{Im } v_i$ ,  $i = 3, 4$  are related to those obtained from  $A_2$  for  $i = 2, 5$  by the same factor.

For the case with longitudinal polarization, the sensitivities for the relevant  $\text{Im } v_i$  are enhanced by almost an order of magnitude, whereas the sensitivities for  $\text{Im } a_i$  are improved marginally. For the case of anomalous triple gauge-boson couplings contributing to the process, a similar conclusion was obtained in [10].

All the results discussed above are now summarized in the Tables 2, 3 and 4.

$A_1$			$A_2$		
Re $v_3$	Re $v_4$	Re $v_6$	Im $v_3$	Im $v_4$	Im $v_6$
$2.1 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$	$4.6 \cdot 10^{-2}$
Re $a_3$	Re $a_4$	Re $a_6$	Im $a_3$	Im $a_3$	Im $a_6$
$3.1 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$	$4.6 \cdot 10^{-2}$	$2.1 \cdot 10^{-4}$	$2.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-3}$

Table 2: Table of sensitivities obtainable at the LC with the machine and operating parameters given in the text for the asymmetries  $A_1$  and  $A_2$ .

$A_3$		
Im $v_2$	Im $v_5$	Im $v_6$
$9.3 \cdot 10^{-4}$	$9.3 \cdot 10^{-4}$	$1.4 \cdot 10^{-2}$
Im $a_2$	Im $a_5$	Im $a_6$
$6.4 \cdot 10^{-5}$	$6.4 \cdot 10^{-5}$	$9.6 \cdot 10^{-4}$

Table 3: Table of sensitivities obtainable at the LC with the machine and operating parameters given in the text for the asymmetries  $A_3$  with unpolarized or transversely polarized beams.

$A_3$		
Im $v_2$	Im $v_5$	Im $v_6$
$5.6 \cdot 10^{-5}$	$5.6 \cdot 10^{-5}$	$8.4 \cdot 10^{-4}$
Im $a_2$	Im $a_5$	Im $a_6$
$5.2 \cdot 10^{-5}$	$5.2 \cdot 10^{-5}$	$7.9 \cdot 10^{-4}$

Table 4: Table of sensitivities obtainable at the LC with the machine and operating parameters given in the text for the asymmetries  $A_3$  with longitudinally polarized beams.

## 5 Conclusions

Forward-backward asymmetry of a neutral particle with polarized beams as a signal of CP violation has been studied here in some generality. We have considered a general form-factor parametrization and have isolated from these (combinations of) CP-violating form factors. Only one out of these corresponding to  $i = 6$  has the special property of contributing to both polarization dependent as well as independent parts of the cross section. Two out of the rest corresponding to  $i = 2, 5$  can have observable consequences in the absence of transverse polarization, while those corresponding to  $i = 3, 4$  can only be studied in the presence of transverse polarization. Since the former ones occur in the asymmetry  $A_3$  which is even under naive time reversal, the CPT theorem implies that in such a case the asymmetry is proportional to the absorptive part of the amplitude. The sensitivities for  $\text{Im } v_i$  ( $i = 2, 5$ ) are improved by an order of magnitude with the use of longitudinal polarization, whereas the sensitivities for  $\text{Im } a_i$  ( $i = 2, 5$ ) are improved only marginally. The asymmetry  $A_1$  that we study in the presence of transverse polarizations includes also an azimuthal angle asymmetry, which makes it odd under naive time reversal. It is thus proportional to the real part of the couplings. This real part cannot be studied without transverse polarization.

In general, one can conclude that longitudinal beam polarization plays a useful role in improving the sensitivity to absorptive parts of CP-violating form factors, which are amenable to measurement even without polarization. However, transverse polarization enables measurement of dispersive parts of certain form factors which are inaccessible without polarization or with longitudinal polarization.

This work extends recent results where CP violation due to anomalous triple-gauge-boson vertices was considered. Anomalous triple-gauge-boson couplings would occur at loop level through triangle diagrams in theories like minimal supersymmetric standard model (MSSM) [16] or multi-Higgs models involving particles beyond SM coupling to gauge bosons. The form factors we consider here include these contributions, as well as additional form factors which might also arise in these theories through box diagrams [17]. It is thus natural to include all form factors. We have shown that with typical LC energies and realistic integrated luminosities and degrees of electron and positron beam polarization, a window of opportunity for the discovery of new physics can be opened.

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